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ANALOG DOT RECORDING AS AN EXEMPLARY PROBLEM IN THE AREA OF DYNAMIC MEASUREMENTS

Analog recorders as typical measuring instruments disappear from contemporary equipment due to progress of digital measurements and common possibilities of measurement data storage in a computer memory. However, recorders can be improved by application of the idea of model-based control. It creates new, even surprising possibilities. These incline to deep considerations.

Keywords: dot recording, dynamic measurement

1. INTRODUCTION

Considering the recorder from the point of view proper for dynamic measurements one can treat it like a device fulfilling the two main tasks: it should process the input electrical signal (voltage or current) into a mechanical effect (linear or angular displacement) which accurately "follows" the input signal assuring an admissible level of dynamical errors. Subsequently, it should record the mechanical output signal on an information carrier (paper strip or disc). The mathematical models of solid line recorders are usually represented by means of a transfer function $K(s)$ or nonlinear differential equations describing the transformation of the input electric signal to a mechanical one. If the actual value of the input signal $x(t)$ has to be recorded, then a "sliding mode" state of the recorder servo-mechanism guarantees lack of dynamical errors [6, 11]. That state of the servo-mechanism can exist, if absolute values of derivatives of $x(t)$ are limited to a suitably small number. For so called "dotted recording" one has the possibility of applying highly-sensitive electromechanical transducers. Furthermore, dedicated algorithms for elimination of dynamical errors can be implemented. The result of dotted recording has to be sufficiently dense (otherwise information is lost) but the necessity of coupling the frequency of dotting with the dynamic properties of the recorder limits the available "density of dotting" [1, 7].

The classic approach assumes that the time interval T_i between the consecutive points (dots) cannot be shorter than the transient state duration time t_u of the recorder moving organ. Under that classic assumption the current operation of dotting does not influence the recording of the next point. However, compliance with the classic approach excludes the possibility of implementation of additional improvements aimed at the reduction of the dynamical error. Thus, the more general assumptions can lead us to quite interesting results.

2. DYNAMICS OF THE RECORDER FOR VARIABLE FREQUENCY OF DOTTING

Considering the operation of a dot recorder we can indicate two repeating stages. During the first stage the recorder moving organ is driven by the exciting input signal $x(t)$ and its movement is described by the respective differential equation. Let us denote the duration time of this "free" movement by T_i. During the second stage, for moments T_i, $2T_i, \ldots, nT_i$, the moving organ is stopped and its current displacement is recorded on an information carrier in form of a dot. The duration time of single operation of dotting can be neglected comparing it to T_i. For linear dynamics of the recorder we can find its impulse response $k(t)$ by referring to the transfer function $K(s)$. Next, using the convolution integral, for $0 \leq t \leq T_i$, one can write [4]:

$$y(nT_i + t) = y(nT_i) + \int_0^t k(v)\ \{x(nT_i + t - v) - y(nT_i)\}\ dv \qquad (1)$$

where $y(nT_i)$, $y(nT_i + t)$ - displacements of the recorder moving organ, $x(nT_i + t)$ - values of the recorder input signal. For $t = T_i$, i.e. for the next dotting, we obtain:

$$y(nT_i + T_i) = y(nT_i)\ [1 - h(T_i)] + \int_0^{T_i} k(v) \cdot x(nT_i + t - v)\ dv. \qquad (2)$$

The expression (2) can be rewritten in simplified form:

$$y(n+1) = y(n)\ [1 - h(T_i)] + \int_0^{T_i} k(v)\ x(n + t - v)\ dv. \qquad (3)$$

Basing on Eq. (2) we can formulate the stability condition for the considered procedure:

$$0 < h(T_i) < 2, \qquad (4)$$

where $h(T_i)$ denotes the value of the recorder step response

$$h(t) = \int_0^t k(v)\ dv, \qquad (5)$$

for $t = T_i$.

Let signal $x(nT_i + t)$ for $0 \leq t \leq T_i$ be given in the form:

$$x(nT_i + t) = x(nT_i) + \frac{x(nT_i + T_i) - x(nT_i)}{T_i}\ t + \sum_{j=1}^{\infty} S(nT_i, j)\ \sin(\pi\ j\ \frac{t}{T_i}), \qquad (6)$$

where $S(nT_i, j)$ represents the amplitude of j-th harmonic. Hence, for the considered time interval one obtains:

$$y(n+1) = y(n)\ \left[1 - h(T_i)\right] + x(n)\ \left[h(T_i) - H(T_i)\right] + x(n+1)\ H(T_i) + \sum_{j=1}^{\infty} S(n,j)\ R(j,T_i), \quad (7)$$

where:

$$H(T_i) = \frac{1}{T_i}\int_0^{T_i} h(v)\ dv\ , \quad R(j,T_i) = \int_0^{T_i} k(v)\ \sin\left[\frac{\pi\ j\ (t-v)}{T_i}\right]\ dv. \quad (8)$$

If

$$h(T_i) = 1\ , \quad H(T_i) = 1\ , \quad R(j,T_i) = 0 \quad \text{for} \quad j = 1, 2, \ldots \infty, \quad (9)$$

then $y(n+1) = x(n+1)$. Thus, if (9) holds then recording is done perfectly (lack of dynamical error). A similar effect exists for $h(T_i) = H(T_i) = 1$, if $S(nT_i, j) = 0$, i.e. for a smooth signal $x(t)$ between points nT_i and $nT_i + T_i$. Obviously, if $x(0) = x_o \neq 0$ then the initial point on carrier $y(0)$ is charged with error but all consecutive records $y(n)$ are realized according to the rules presented above.

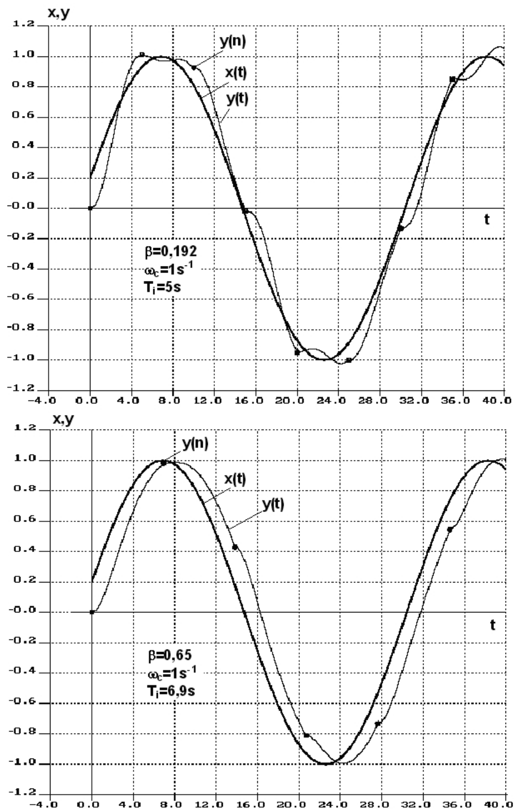

Fig. 1. Signals $x(t)$, $y(t)$, $y(n)$ for $x(t) = \sin[0.2(t+1)]$.

If dynamics of the recorder can be represented by transmittance:

$$K(s)=\frac{1}{1+2\ B\ \dfrac{s}{\omega_0}+\left(\dfrac{s}{\omega_0}\right)^2}, \tag{10}$$

where B- damping coefficient, ω_0- natural, angular velocity, then conditions $h(T_i)=1$ and $H(T_i)=1$ yield the following equations:

$$\omega_0 T_i=\frac{2r\pi-\arccos B}{\sqrt{1-B^2}}$$
$$r=\frac{1}{2\pi}\left(\arccos B-\frac{\sqrt{1-B^2}}{B}\ln 2B\right), \tag{11}$$

which should be fulfilled for positive integer r. Putting $r = 1$ one obtains $B = 0.192$, $(\omega_0 T_i) \cong 5.0$. Solving (11) for $r = 2$ we obtain $B = 0.124$, $(\omega_0 T_i) \cong 11.2$. Generally, the bigger the value r the smaller B and bigger $\omega_0 T_i$. This means, that the solution for $r = 1$ can be regarded as the most advantageous. For values B and T_i referring to $r = 1$ one obtains: $R(1, T_i) \cong 0.92$, $R(2, T_i) \cong -1.3$, $R(3, T_i) \cong - 0.4$, $R(4, T_i) \cong -0.2$ and so on. It can be observed that values of further coefficients R decrease, even rapidly. Thus, the solution is sensitive to "non-smoothness" of the input signal $x(t)$ because of the small value of B. Under the classic assumptions, for $B = 0.65$, the condition $h(T_i) = 1$ is fulfilled if $(\omega_0 T_i) = 6.91$. Thus, for a longer (about 37%) time interval between neighboring points is $H(T_i) \neq 1$ and the dynamical errors are present, even for smooth $x(t)$. However, for the above "classic" conditions is $R(1, T_i) \cong 0.62$, $R(2, T_i) \cong -0.85$, $R(3, T_i) \cong 0.35$, $R(4, T_i) \cong -0.22$. Hence we can conclude that the solution is less sensitive to non-smoothness of signal $x(t)$. The signals $x(t)$, $y(t)$ and recorded points $y(n)$ are shown in Fig. 1. The presented results have been obtained for $x(t) = \sin[0.2(t + 1)]$, $\omega_0 = 1s^{-1}$, $B = 0.65$, $T_i = 6.9$s (classic recording) and for $B = 0.192$, $T_i = 5$s. Analyzing the second variant we observe that dynamical errors appear for those fragments of the record, where signal $x(t)$ becomes a "non-smooth" one (neighborhood of its extreme values). The above observation confirms that the presented evaluations are correct. We should pay attention to an important disadvantage of non-classic solution: the simultaneous recording of several input signals on one carrier is not available.

3. THE MODEL-BASED CONTROL OF RECORDING

Recording $y(t)$ at time moments where

$$x(t) = y(t), \tag{12}$$

should give a perfect result, i.e. total elimination of dynamical errors. The release of a short impulse stopping the recorder moving organ and putting into operation a dotting device (pen lift) if $x(t)$ - $y(t)$= 0 is technically feasible by means of a proper electronic system. However, the signals under comparison $x(t)$, $y(t)$ have got a different physical "nature" ($x(t)$ - current or voltage, $y(t)$ - linear or angular displacement). That is why an electronic model of recorder dynamics has to be applied. The structure of the model can be composed of integrators, adders and non-linear static converters and the system structure design can imitate models realized by means of analog computers [3]. The electronic model of the recorder should be supplied with voltage $x(t)$. Thus, the model output can be used for detection of these time moments where condition (12) holds. Of course, the operation of dotting should reset the respective integrators belonging to the electronic model of recorder (note that some state variables of the "real device" are also reset when its moving organ is stopped). The high accuracy of the recorder model assures the correct operation of recorder but we should take into consideration other circumstances. Because of recorder dynamical errors the states defined by Eq. (12) are attained rather rarely. To overcome this difficulty one should apply an additional "sweeping signal" to the recorder and its model inputs. For example, the sweeping signal can be chosen in the form:

$$x_d(t) = A \operatorname{sgn}(\sin \omega_d t). \tag{13}$$

The choice of amplitude A and angular frequency ω_d ought to guarantee that the recorder response $y(t)$ will not exceed the measuring range, the condition (12) will be fulfilled frequently and the thermal overload of recorder will not appear. The recorder and its model can be controlled with the sum of signals $x(t) + x_d(t)$, or even with a single signal $x_d(t)$. In this second case $x_d(t)$ should excite the response $y(t)$ covering the whole measuring range. If $x_d(t)$ is given in the form:

$$x_d(t) = A \ \sin(\omega_d \ t), \tag{14}$$

the measuring range is $\pm\, y_{max}$ and the recorder transfer function is $K(j\omega)$, then condition:

$$A \ \left|K(j\omega_d)\right| = y_{\max}, \tag{15}$$

has to be fulfilled. Defining substantively the permanent thermal overload of recorder by means of the admissible amplitude of signal A_d the additional condition (16) should be attached to the previous one (15):

$$\frac{y_{\max}}{\left|K(j\omega_d)\right|} = A_d. \tag{16}$$

Using (16) one can determine the maximum value of the angular frequency ω_d. The described operation of the recorder is close to the idea of operation of so-called "evolving systems" [5]. Despite of essential differences the idea of recorder operation can also be treated as being more

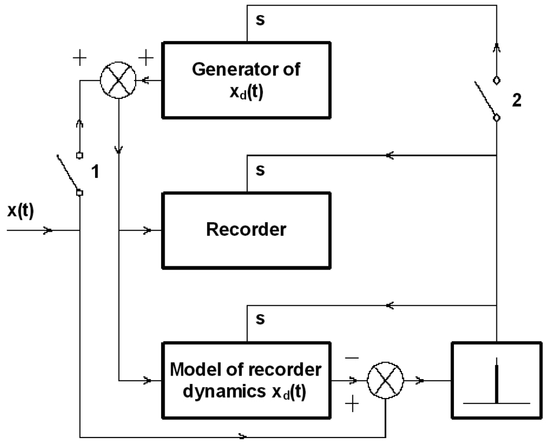

Fig. 2. The block diagram of a model based dot recorder with an additional generator of auxiliary signal $x_d(t)$.

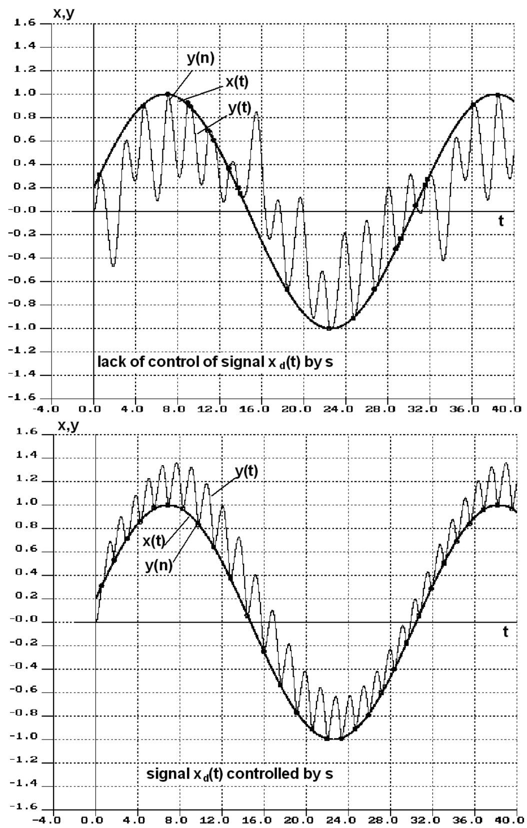

Fig. 3. Signals $x(t)$, $y(t)$, $y(n)$ for $x(t) = \sin[0.2(t + 1)]$, $x_d(t) = 3\cos3t$, if control is based on sum $x(t) + x_d(t)$ and recorder parameters are as follows: $\omega_0 = 1s^{-1}$, $\beta = 0.65$.

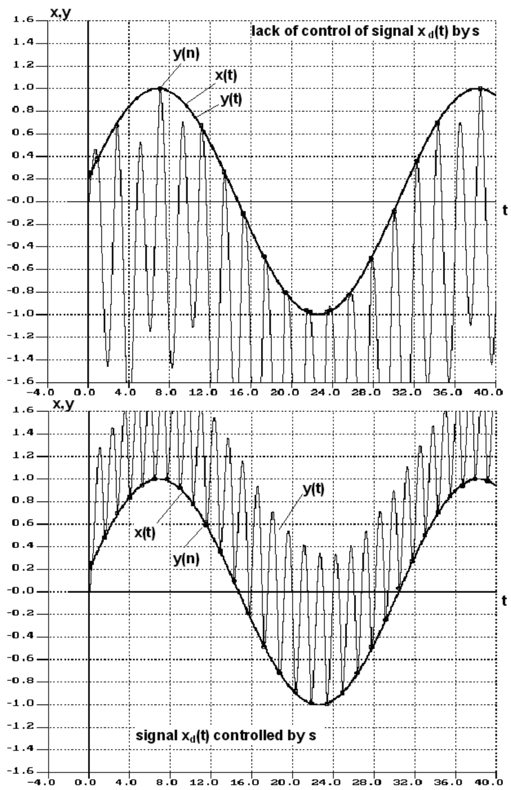

Fig. 4.Signals $x(t)$, $y(t)$, $y(n)$ for $x(t) = \sin[0.2(t + 1)]$, $x_d(t) = 3\cos 3t$, if control is realized by the use of $x_d(t)$ and recorder parameters are as follows: $\omega_0 = 1s^{-1}$, $\beta = 0.65$.

or less similar to the operation of Keinath's graphic compensator [2]. For nonlinear dynamics of the recorder and its model (representation by means of nonlinear differential equations) formulae (15) and (16) are not valid. The introduction of periodical switching of measuring channels makes that simultaneous recording of several signals on one document (information carrier) can be realized as well (like in the case of classic recorders). The block diagram of a recorder operating according to the above principles is shown in Fig. 2, however the structure realizing periodical switching of measuring channels has been neglected. For $x(t) = y(t)$ signal s brings to zero the recorder and model initial conditions $y^{(i)}(t)$, for $I = 1, 2, \ldots$, n. It can be also used (if switch 2 is "on") to periodical initiation of generation of signal $x_d(t)$. In such case the same values of initial conditions for generation of signal $x_d(t)$ should be repeated. Putting the switch 1 to position "on" one causes that the recorder and its model are controlled by the sum of signals $x(t) + x_d(t)$. Thus, there are four combinations of system behavior.

The exemplary signals $x(t)$, $y(t)$, $y(n)$ for $x(t) = \sin[0.2(t + 1)$ and $B = 0.65$, $\omega_0 = 1s^{-1}$ are shown in Figs. 3 and 4. If switch 1 is "on" then $x_d(t) = 3\cos(3t)$. If switch 1 is "off" then $x_d(t) = 9\cos(3t)$. Setting identically the initial conditions of the generator of signal $x_d(t)$ after every dotting operation (switch 2 in position "on") we obtain the better, more uniform distribution of density of recorded points on the information carrier. If we want to minimize the thermal overload of the recorder, then switch 1 should be in position "on", which means that the sum $x(t) + x_d(t)$ is used for control of generator of $x_d(t)$. Of course, both of the above statements are true if dynamic errors of the recorder for input $x(t)$ are not too great.

4. THE APPROXIMATE FORMULAE

The use of additional signal $x_d(t)$ makes that the frequency of dotting significantly increases. This allows to apply the approximate models, those obtained for slowly-changing input signals, if the convolution integrand can be expanded in Taylor series [9]:

$$y(t) = \int_0^t k(v)\ x(t-v)\ dv$$
$$x(t-v) = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} v^i\ x^{(i)}(t). \tag{17}$$

The certainly useful, simplified model of this type $y_m(t)$ takes the form:

$$y_m(t) = h(t)\ x[t - t_o(t)], \tag{18}$$

where $h(t)$ represents the step response for transfer function $K(s) \mathrel{\widehat{=}} k(v)$ and:

$$t_o(t) = h^{-1}(t) \int_0^t v\ k(v)\ dv \quad .$$

For transfer function (10) and small t one obtains $t_o(t) = 2/3t$. Now, using model (18) we obtain the simplified form of expression (1):

$$y(t_i + t) = x(t_i)[1 - h(t)] + h(t)\left\{x\left(t_i + \frac{t}{3}\right) + x_d\left(\frac{t}{3}\right)\right\}, \tag{19}$$

if $x_d(t)$ is generated from the beginning for every one consecutive cycle (control by means of sum $x(t) + x_d(t)$), or in the form;

$$y(t_i + t) = x(t_i)\ [1 - h(t)] + h(t)\ x_d\left(\frac{t}{3}\right), \tag{20}$$

for control realized by means of $x_d(t)$. For both cases (formulae (19), (20)) t_i denotes the moment of signal recording (dotting), where $y(t) \approx x(t)$. The next record is done at moment t where:

$$y(t_i + t) = x(t_i + t) = x(t_i) + t\ x^{(1)}(t_i). \tag{21}$$

Hence, after simple transformations we can write:

$$t\ x^{(1)}(t_i)\ \{1-h(t)\}=h(t)\ x_d\left(\frac{t}{3}\right) \tag{22}$$

using formula (19), or :

$$x(t_i)\ h(t)+t\ x^{(1)}(t_i)=h(t)\ x_d\left(\frac{t}{3}\right) \tag{23}$$

using expression (20). So, taking into consideration the expressions given above, we can see that the frequency of dotting increases for these time intervals where the derivative of the recorded signal $x^{(1)}(t)$ is close to zero and results of experiments confirm this conclusion (see Figs. 3 and 4). The inaccuracies of expressions (22) and (23) are caused by the assumption that $x_d(t)$ fulfils completely the requirements for slowly-altering signals. This does not fit to real conditions. That is why addends $h(t)x_d(t/3)$ in (22) and (23) should be substituted with either convolution of signal $x_d(t)$and impulse response $k(v)$ or the approximate formula:

$$\int_0^t x_d(v)\ k(t-v)\ dv \approx k(t)\ \int_0^t x_d(v)\ dv, \tag{24}$$

representing the other simplified model [20]. The formulae given above hold, if signal $x_d(t)$ is controlled by signal s and generator of $x_d(t)$ always starts (after dotting) from the identical initial conditions. This has been mentioned, that other solution seems to be disadvantageous and simplified formulae become very complicated. For this reason these are not quoted. The models describing the recorder operation in case of "simultaneous" (to be more precise we should say "one by one") recording of several input signals on a common carrier are more complicated. It is obvious that for such way of recording we obtain bigger distances between consecutive

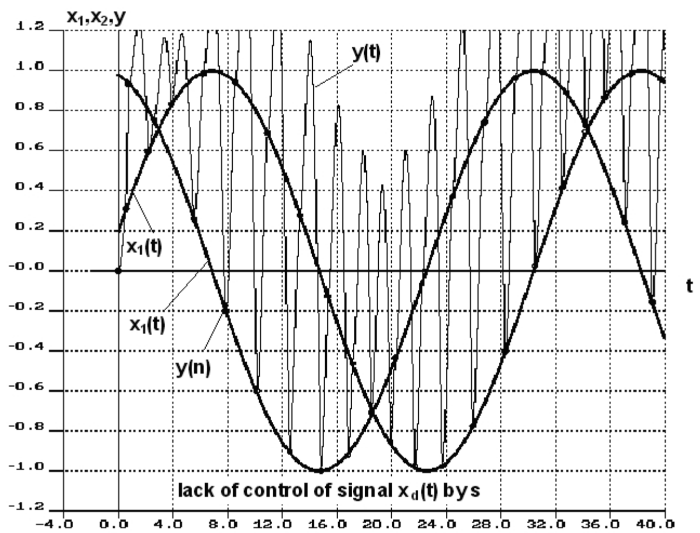

Fig. 5. Signals $x_1(t)$, $x_2(t)$, $y(t)$, $y(n)$ for $x_1(t)$ = sin[0.2(t + 1)], $x_2(t)$ = cos[0.2(t + 1)], $x_d(t)$ = 9cos3t, if control is realized by use of sum $x(t) + x_d(t)$ and recorder parameters are as follows: $\omega_0 = 1s^{-1}$, $\beta = 0.65$.

points of chart $y_1(n)$, $y_2(n)$, ... and those distances depend on mutual relations between signals $x_1(t), x_2(t), \ldots$. The situation for two input channels, if $x_1(t) = \sin[0.2(t1)]$ and $x_2(t) = \cos[0.2(t + 1)]$, for identical signals $x_d(t) = 9\cos(3t)$, is illustrated in Fig. 5. During the experiment under consideration the signal $x_d(t)$ was not controlled by signal s.

5. OTHER SOLUTIONS

Considering other solutions we can repeat the previous statement about the possibility of application of another electromechanical device with nonlinear dynamics, namely, it can be a servomechanism operating in the state called the "sliding mode". Dependencies such as those given above are not valid now. As an essential innovation one can treat the modification of generator control of $x_d(t)$ by means of s: the signal x_d should increase, for example, according to the expression $x_d(t_i + t) = kt$, while $x(t_i + t)$ increases and it should be decreasing (e.g. $x_d(t_i + t) = -kt$) if $x(t_i + t)$ decreases. A solution of this type would be associated with the necessity of using a memory block or storage of consecutive values of the recorded signal. The generator for that application could be a crelatively simple device (basing on controlled integrators). Moreover, the possibility of exceeding the measurement range would be eliminated.

6. SUMMARY

The possibilities of improvement of classic electromechanical transducers used as dot recorders seem to be exhausted due to frequency limits for recording caused by transducer dynamical properties. The described applications of the idea of model-based control and introduction of an auxiliary generator of signal $x_d(t)$ substantially decrease the recorder dynamic errors and increase the frequency of recording. The specific thermal overload of the electromechanical transducer seems to be the main restriction influencing the possible range of improvement.

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